Kalman Filter in Attitude Determination

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1 Kalman Filter
   - Introduction
   - Applications

2 Vehicle’s Attitude
   - Euler Angles
   - Quaternions

3 Mathematical Formulation
   - Dynamic System Model
   - Kalman Filter
     - Kalman Filter Algorithm
     - Filter Equations

4 Conclusion & Final Remarks
One of the primary developers was Rudolph Emil Kálmán (May 19, 1930 - July 2, 2016).

It’s an optimal estimation algorithm used to estimate states of a system from indirect and uncertain measurements.

### Kalman Filter Applications

- Avionics
- Navigation Systems
- Control Systems
- Trajectory optimization
Kalman Filter Applications
Attitude Representation

Attitude Rate in Body Frame

- Roll rate - $p = \dot{\phi}$
- Pitch rate - $q = \dot{\theta}$
- Yaw rate - $r = \dot{\psi}$

Attitude Rate in Reference Frame

$$\dot{\phi} = p + (q \sin \phi + r \cos \phi) \tan \theta$$

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\psi} = \frac{q \sin \phi + r \cos \phi}{\cos \theta}$$

Quaternions

$$\vec{\eta} = \eta_0 + \eta_1 \vec{i} + \eta_2 \vec{j} + \eta_3 \vec{k}$$
Mathematical Formulation

Dynamical System Model

\[
\dot{x}(t) = A(t)x(t) + B(t)u(t) + v(t) \\
y(t) = C(t)x(t) + w(t)
\]

- \( A \) - State transition model
- \( x \) - State vector
- \( B \) - Control input model
- \( u \) - Control vector
- \( v \) - Process noise
- \( y \) - Measurements vector
- \( C \) - Observation model
- \( w \) - Measurements noise

Vehicle’s Attitude

\[
x = \begin{bmatrix}
\theta \\
\phi \\
\psi
\end{bmatrix} \vee \vec{\eta}
\]

\[
u = \begin{bmatrix}
\delta_0 \\
\delta_1 \\
\vdots \\
\delta_n
\end{bmatrix}
\]

\[
y = \begin{bmatrix}
q \\
p \\
r
\end{bmatrix} \quad \text{(Example)}
\]
Kalman Filter Considerations

- **Process Noise** $v(t)$
  - $v \sim N(0, Q_k)$
  - There is no correlation between $v(t_i)$ and $v(t_k)$.

- **Measurements Noise** $w(t)$
  - $w \sim N(0, R_k)$
  - There is no correlation between $w(t_i)$ and $w(t_k)$.

The Kalman filter should not be used if these considerations do not apply to the system subjected to a filtering process.
Mathematical Formulation

Prior knowledge of state

Next timestep
\[ k = k + 1 \]

Estimate of State
\[ \hat{x}_k^+ \]
\[ P_k^+ \]

Prediction Step
(Physical Model)
\[ \hat{x}_k^- \]
\[ P_k^- \]

Update Step
Comparison between prediction and measurements

Measurements \( y_k \)
Mathematical Formulation

Prediction

\[ \hat{x}_k^- = A_{k-1} \hat{x}_{k-1} + B_{k-1} u_{k-1} \]
\[ P_k^- = A_{k-1} P_{k-1}^+ A_{k-1}^T + Q_k \]

Update

\[ K_k = P_k^- C_k^T (C_k P_k^- C_k^T + R_k)^{-1} \]
\[ \hat{x}_k^+ = \hat{x}_k^- + K_k (y_k - C_k \hat{x}_k^-) \]
\[ P_k^+ = (I - K_k C_k) P_k^- \]

\[ \lim_{P_k^- \to 0} K_k = 0 \Rightarrow \hat{x}_k^+ = \hat{x}_k^- \]
\[ \lim_{R_k \to 0} K_k = C_k^{-1} \Rightarrow \hat{x}_k^+ = C_k^{-1} y_k = y_k \]
The Kalman filter is an optimal estimation algorithm used to estimate states of a system. It is capable of estimating variables that were not measured or measured indirectly, using the physical model. The algorithm is capable of fusing the predictions and measurements based on their uncertainty. The filter can be implemented in a controller in order to develop, for example, a LQG (Linear Quadratic Gaussian).
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